Tensionless structure of a glassy phase

A. Lipowski[1] and D. Johnston[2]
[1]Department of Physics, A. Mickiewicz University, 61-614 Poznań, Poland
[2]Department of Mathematics, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom
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We study a class of homogeneous finite-dimensional Ising models which were recently shown to exhibit glassy properties. Monte Carlo simulations of a particular three-dimensional model in this class show that the glassy phase obtained under slow cooling is dominated by large-scale excitations whose energy E_l scales with their size l as $E_l \sim l^{\Theta}$ with $\Theta \sim 1.33(20)$. Simulations suggest that in another model of this class, namely the four-spin model, energy is concentrated mainly in linear defects, making the domain walls tensionless in this case also. Two-dimensional variants of these models are trivial and most likely the energy of excitations scales with the exponent $\Theta = 1$.

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Recently the problem of the structure of the glassy phase in spin glasses has attracted considerable attention. The main thrust of research has been to establish whether the low-temperature phase is described by the so-called droplet model [1] or by the replica symmetry breaking (RSB) theory of Parisi [2]. An important difference between these theories concerns the energy E_l of large-scale excitations, which should scale with their size l as $E_l \sim l^{\Theta}$ where $\Theta > 0$ for the droplet model but with $\Theta = 0$ for the RSB theory. The most interesting situation arises in the three-dimensional case, where it seems that a combination of these two approaches is needed to describe the glassy phase correctly [3].

Of course, the problem of the structure of the glassy phase is not restricted to spin glasses. Disordered, out of equilibrium, slow dynamics structures appear in superconducting compounds, polymers, and granular matter. However, these systems are very complex and modeling them seems to be more difficult than understanding spin glasses. Conventional glasses are also very complex [4]. Nevertheless, recently relatively simple models have been proposed which exhibit an encouraging number of glassy properties [5–9]. All these models are spin models that do not contain quenched disorder as in the case of spin glasses and glassiness is dynamically generated. An absence of quenched disorder has important implications. First, the ground state and sometimes even the structure of excitations are known exactly. Let us emphasize that for the three-dimensional spin glasses the problem of finding the ground state is extremely difficult (nonpolynomial complete) and is one of the main difficulties in numerical approaches to spin glasses. Secondly, for models without quenched disorder there is no need to average over different realizations of this disorder, which is yet another computational advantage of such models.

The objective of the present paper is to examine the nature of the glassy phase in certain nondisordered Ising models. Simple heuristic arguments show that these models might have large-scale excitations of energy that scale with their size as l^{d-2}, i.e., slower than the surface $(\sim l^{d-1})$. The question is whether such states appear in, or maybe even dominate, the glassy phase. To examine this problem we have performed Monte Carlo simulations of the models and our results suggest that the glassy phase is dominated by excitations whose energy increases faster than l^{d-2} but slower than l^{d-1}. It is also likely that the exponent Θ that describes the size dependence of the energy of excitations takes some nontrivial values for these models. For the models of conventional glasses considered here the problem of energetics of large-scale excitations is computationally much more tractable than for spin glass models. It is hoped that the results obtained for these models will provide some insight into other glassy systems too. In addition, our results can be used to verify some earlier claims concerning the nature of the glassy transition in some of these models [10].

The class of models that we examine in this paper is defined by the following Hamiltonian:

$$H = -2k \sum_{\langle i,j \rangle} S_i S_j + \frac{k}{2} \sum_{\langle\langle i,j \rangle\rangle} S_i S_j + \frac{(1-k)}{2} \sum_{[i,j,k,l]} S_i S_j S_k S_l , \tag{1}$$

where the summations are over nearest neighbors, next-nearest neighbors, and elementary plaquettes, respectively. Model (1) has the interesting property that the energy of certain excitations of size l is proportional to l^{d-2}, whereas typically the energy of an excitation for a standard nearest neighbor Ising model is proportional to its surface $(\sim l^{d-1})$. This property has been used to construct a class of random surface theories based on model (1) [11].

Recently, it was shown that for $d=3$ model (1) has slow dynamics at low temperature [10]. When a high-temperature sample is quenched to low temperature the excess energy $\delta E = E(t) - E_{eq}$, where E_{eq} is the equilibrium energy, decays with time t much more slowly than $t^{-1/2}$, which is a typical decay rate for Ising models with nonconservative dynamics [12]. It turns out that the $k=0$ (pure four-spin interaction) case is of particular interest. This is because in this case the model also has some other properties typical of conventional glasses, such as strong metastability [6] and small cooling-rate effects [13]. Moreover, certain time dependent correlation functions, such as those describing aging, also behave similarly to real glassy systems [14]. Although a slow decay of δE is an indication of slow dynamics, it would be desirable to relate this decay to the increase of a characteristic length scale l. (As we will see, such a relation will give some information about the energetics of excitations of the glassy phase.)

For ordinary Ising models simple arguments, based on the fact that the energy of excitation of size l scales as its surface ($E_l \sim l^{d-1}$), can be used to obtain the relation

$$\delta E \sim \frac{1}{l}. \tag{2}$$

However, since for model (1) the energy of excitations might increase more slowly than their surface area, the relation (2) is no longer obvious. Assuming that in the glassy phase the dominant excitations are indeed these low-energy excitations (with $E_l \sim l^{d-2}$), the following relation should hold [10,13]:

$$\delta E \sim \frac{1}{l^2}. \tag{3}$$

Let us note that the assumption that the glassy phase is composed of low-energy excitations implies that at the glassy transition domain walls lose their surface tension. Such an identification might be of more general validity and could be used as a criterion to locate the glassy transition.

Is it possible to verify which of the relations (2) and (3) is true in our model? First, let us note that (2) and (3) are two extremal cases corresponding to the largest and the smallest excitation energy per surface area, respectively. It is thus possible that neither of them is true and in the glassy phase an intermediate relation holds. To consider a more general situation let us assume that the energy of excitations scales as l^{Θ}. In a lattice of linear size L the number of excitations of size l scales as $(L/l)^d$ and the total excess energy scales as $(L/l)^d l^{\Theta}$. Thus, the excess energy per spin δE scales as

$$\delta E \sim l^{\Theta - d}. \tag{4}$$

To determine Θ we need a second, independent measurement of the characteristic length l. It is already known that l can also be obtained from the fluctuations of the order parameter using the relation [15]

$$\chi = \frac{1}{L^d}\left\langle \frac{1}{L^d}\left(\sum_i S_i\right)^2\right\rangle = l^d, \tag{5}$$

where the magnetization is taken as a corresponding order parameter. Assuming that relation (5) determines the same characteristic length as Eq. (4) (up to the order of magnitude), we use the following procedure. We continuously cool the high-temperature sample down to zero temperature. In this process the temperature changes linearly with simulation time,

$$T(t) = T_0 - rt, \tag{6}$$

where r is the cooling rate. Then, for the zero-temperature configuration we calculate δE and χ. Previous Monte Carlo simulations suggest that for model (1) and $k=2$ the zero-temperature energy excess δE decreases with r as

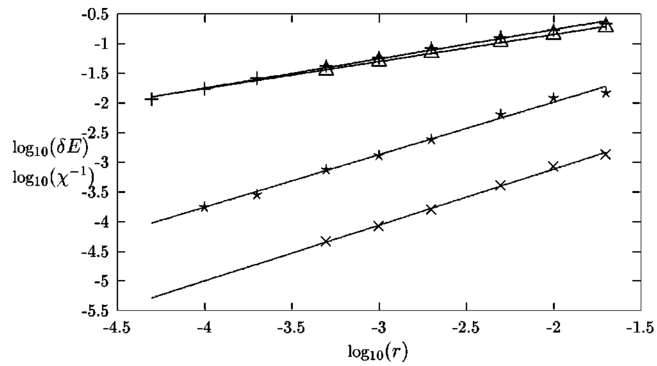

FIG. 1. The excess energy δE ($+$, $d=3$, and $\triangle$, $d=2$), and χ^{-1} ($\star$, $d=3$, and $\times$, $d=2$) as a function of the cooling rate r.

$$\delta E \sim r^{x_1}, \tag{7}$$

where $x_1 = 0.50(5)$ [13]. Similarly, we expect that χ also increases as a power of r,

$$\chi \sim r^{-x_2}. \tag{8}$$

Inverting Eq. (8) and using Eq. (5) we obtain $r \sim l^{-d/x_2}$ and from Eq. (7) we have $\delta E \sim l^{-dx_1/x_2}$. Finally, comparing the last relation with Eq. (4) we obtain

$$\Theta = d(1 - x_1/x_2). \tag{9}$$

To estimate Θ we performed Monte Carlo simulations of model (1) for $k=2$ and using a Metropolis algorithm [16] with random update. Simulations were made for the system size up to $L=70$ and we have checked that this is sufficient to obtain basically size independent results. For each cooling rate r we made around 100 independent runs which were used to calculate δE and χ. The starting temperature was $T=2.8$, which for $k=2$ is above the critical temperature, which in this case is $T_c \sim 2.35$ [10].

The results of our simulations are shown in Fig. 1. The relatively good linearity of our data confirm the power-law behavior (7) and (8). From these data we estimate $x_1 = 0.50(5), x_2 = 0.90(5)$, and using Eq. (9) we obtain $\Theta = 1.33(20)$. Such an estimate of Θ shows that neither Eq. (2), which corresponds to $\Theta = 2$, nor Eq. (3), which corresponds to $\Theta = 1$, is correct. Instead, we have an intermediate possibility with a noninteger value of Θ. (Marginally, our estimation can be consistent with $\Theta = 1$.) Let us note that since $\Theta < 2$ the surface tension of domain walls vanishes [17].

For comparison, in Fig. 1 we also present results of our simulations for the two-dimensional (square lattice) version of model (1) with $k=2$ (in this case $T_c = 0$). Simulations were performed for system sizes up to $L=1000$. From these data we estimate $x_1 = 0.46(5)$, $x_2 = 0.95(5)$, and thus $\Theta = 1.05(10)$. It is likely that in this case $\Theta = 1$, which would indicate the trivial nature of domain walls with energy proportional to their perimeter, the typical two-dimensional Ising model behavior.

Although for $k=2$ model (1) has slow low-temperature dynamics, it does not display a genuine glassy transition. As

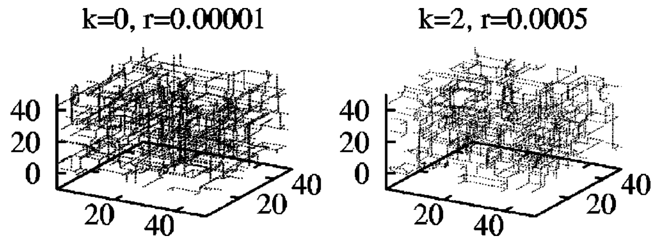

FIG. 2. The distribution of unsatisfied plaquettes in the zero-temperature glassy phase. Simulations were made for the system size $L=100$. (Only a portion of the system is shown.) Let us note that although the cooling for the $k=2$ case is faster it seems to create larger domains. It was already suggested that for the case $k=0$ the model should order much more slowly than for $k=2$ [10].

we already noted, to model glassy transitions one should really study the case of $k=0$. However, in this case the above method encounters some difficulties since the domains of random quench are not only of ferromagnetic type as in the case of $k=2$, but also antiferromagnetic and even of some mixed types (see [10] for some discussion). For $k=0$ Eq. (5) cannot be used and thus the exponent Θ cannot be determined using the above method.

To get some insight into the $k=0$ case we instead looked at the distribution of unsatisfied plaquettes [18] in the glassy phase (i.e., plaquettes contributing energy above the ground state). The random high-temperature sample was slowly cooled down to zero temperature. Then for the final configuration we located unsatisfied plaquettes, and their spatial distribution is shown in Fig. 2. For comparison we also present similar calculations for the $k=2$ case. One can see that in both cases energy is concentrated in linear segments. For $k=2$ this is in agreement with our estimation $\Theta<2$ as for $\Theta=2$ energy would be localized on two-dimensional surfaces. Although for $k=0$ we cannot estimate Θ, the linear stuctures in Fig. 2 strongly suggests that in this case also $\Theta<2$ and the glassy phase is composed of tensionless domain walls.

Of course, the glassy phase obtained by finite-rate cooling contains some regions other than linear segments where energy is concentrated. However, we expect that such spots will diminish for decreasing cooling rate r. To confirm our

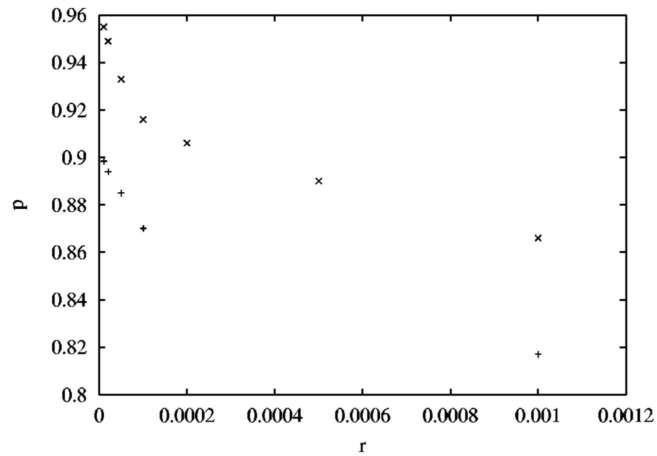

FIG. 3. The fraction p of unsatisfied plaquettes that are located in linear segments as a function of the cooling rate r for $k=0$ $(+)$ and $k=2$ $(\times)$.

expectations we measured the ratio p of unsatisfied plaquettes that belong to linear segments compared to the total number of unsatisfied plaquettes [19]. The results, presented in Fig. 3, show that p indeed increases for decreasing r. It is also possible that in the limit $r\rightarrow 0$ the fraction $p\rightarrow 1$. Let us note that the glassy state obtained in such a limit constitutes an ideal glass [20] and the present results might shed some light on this, so far hypothetical, state of matter. In particular, they suggest that in the ideal glass slow cooling removes energy-rich spots and leaves only low-energy excitations.

In conclusion, we studied the glassy phase of the gonihedric model. Our results show that the energy of excitations in this phase scales as l^{Θ} with $\Theta<d-1$. This confirms earlier expectations that domain walls in this model are tensionless. Since the $k=0$ case seems to have a number of properties typical of ordinary glasses, it would be desirable to check whether this result also has some analogy in real systems.

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[16] See, e.g., K. Binder, in *Applications of the Monte Carlo Method in Statistical Physics*, edited by K. Binder (Springer, Berlin, 1984).
[17] To check our procedure we performed simulations also for the nearest neighbor three-dimensional Ising model. Rough estimations of x_1 (~ 0.5) and x_2 (~ 1.5) based on these simulations suggest that $\Theta = 2$, which is the expected result.
[18] For $k=0$ a plaquette is called unsatisfied if the product of spin variables on this plaquettes equals 1. Let us note that for $k=0$ the coupling in model (1) is positive while that used, e.g., in [6,13] is negative. However, using a simple transformation, the two cases can be made equivalent.
[19] A plaquette separates two elementary cubes. This plaquette belongs to the linear segment when each of the adjacent cubes contains only two unsatisfied plaquettes (including the plaquette under consideration).
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